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ETOOS Comprehensive Study Material For JEE- Main & Advanced

## **DEFINITION**

A relation R from a set A to a set B is called a function if each element of A has unique image in B. It is denoted by the symbol.

**FUNCTION** 

 $A \xrightarrow{f} B$  $f: A \rightarrow B$ or

which reads f' is a function from A to B 'or' f maps A to B,

If an element  $a \in A$  is associated with an element  $b \in B$ , then b is called 'the f image of a' or 'image of a under f 'or' the value of the function f at a'. Also a is called the pre-image of b or argument of b under the function f. We write it as

b = f(A)or  $f: a \rightarrow b \text{ or } f: (a, b)$ 

Thus a function 'f' from a set A to a set B is a subset of  $A \times B$  in which each 'a' belonging to A appears in one and only one ordered pair belonging to f.

**ETOOS KEY POINTS** 

Every function from  $A \rightarrow B$  satisfies the following conditions. ()  $f \subset A \times B$ (ii)  $\forall a \in A \Rightarrow (a, f(A)) \in f$  and (iii)  $(a, b) \in f \& (a, c) \in f \Rightarrow$ b = c

#### **REPRESENTATION OF FUNCTION**

**Ordered pair**: Every function from  $A \rightarrow B$  satisfies the following conditions: **(A)** 

(i)  $f \subset A \times B$  (ii)  $\forall a \in A$  there exist  $b \in B$  and (iii)  $(a, b) \in f \& (a, c) \in f \Rightarrow b = c$ 

 $f: \mathbb{R} \rightarrow \mathbb{R}, y = f(x) = 4x, f(x) = x^2$ (uniformly defined) (i)  $f(x) = \begin{cases} x+1 & -1 \le x < 4 \\ -x & 4 \le x < 7 \end{cases}$ **(ii)** (non-uniformly defined)

(iii) 
$$f(x) = \begin{cases} x^2 & x \ge 0 \\ -x - 1 & x < 0 \end{cases}$$

(non-uniformly defined)

0

#### **(C) Graphical representation :**



Graph(1)

Graph (2)

Graph(1) represent a function but graph(2) does not represent a function.

#### Domain, Co-domain & Range Of A Function

Let  $f: A \rightarrow B$ , then the set A is known as the domain of f & the set B is known as co-domain of f. The set of f images of all the elements of A is known as the range of f.

Thus: Domain of  $f = \{a \mid a \in A, (a, f(A)) \in f\}$ Range of  $f = \{f(A) \mid a \in A, f(A) \in B\}$ 



#### **ETOOS KEY POINTS**

- (i) If a vertical line cuts a given graph at more than one point then it can not be the graph of a function.
- (ii) Every function is a relation but every relation is not necessarily a function.
- (iii) It should be noted that range is a subset of co-domain.
- (iv) If only the rule of function is given then the domain of the function is the set of those real numbers, where function is defined. For a continuous function, the interval from minimum to maximum value of a function gives the range

#### **METHODS OF DETERMINING RANGE**

If y = f(x), try to express as x = g(y), then domain of g(y) represents possible values of y, which is range of f(x).

**Ex.** Find the range of 
$$f(x) = \frac{x^2 + x + 1}{x^2 + x - 1}$$

Sol. 
$$f(x) = \frac{x^2 + x + 1}{x^2 + x - 1} \{x^2 + x + 1 \text{ and } x^2 + x - 1 \text{ have no common factor}\}$$

$$y = \frac{x^2 + x + 1}{x^2 + x - 1}$$
  

$$\Rightarrow \qquad yx^2 + yx - y = x^2 + x + 1$$
  

$$\Rightarrow \qquad (y - 1)x^2 + (y - 1)x - y - 1$$

If y = 1, then the above equation reduces to -2 = 0. Which is not true.

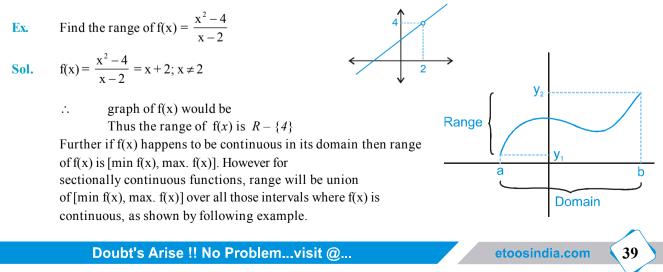
= 0

Further if  $y \neq 1$ , then  $(y-1)x^2 + (y-1)x - y - 1 = 0$  is a quadratic and has real roots if

$$(y-1)^2 - 4 (y-1) (-y-1) \ge 0$$
  
i.e. if  $y \le -3/5$  or  $y \ge 1$  but  $y \ne 1$   
Thus the range is  $(-\infty, -3/5] \cup (1, \infty)$ 

#### (ii) Graphical Method

The set of y- coordinates of the graph of a function is the range.



#### • Etoos Tips & Formulas •

#### 1. Definition

If to every value (considered as real unless other-wise stated) of a variable x, which belongs to a set A, there corresponds one and only one finite value of the quantity y which belong to set B, then y is said to be a function of x and written as  $f: A \rightarrow B$ , y = f(x), x is called argument or independent variable and y is called dependent variable.

Pictorially:  $x \quad f \quad f(x) = y$ 

y is called the image of x & x is the pre-image of y, under f. Every function  $f: A \rightarrow B$  satisfies the following conditions. (i)  $f \subset A \times B$ (ii)  $\forall a \in A \quad \exists b \in B \text{ such that } (a,b) \in f \text{ and}$ (iii) If  $(a,b) \in f \& (a,c) \in f \implies b = c$ 

#### 2. Domain, Co-Domain & Range of a Function

Let  $f: A \rightarrow B$ , then the set A is known as the domain of 'f' & the set B is known as co-domain of 'f'. The set of all f images of elements of A is known as the range of 'f'. Thus

Domain of  $f = \{ x | x \in A, (x, f(x)) \in f \}$ Range of  $f = \{ f(x) | x \in A, f(x) \in B \}$ range is a subset of co-domain.

#### 3. Important Types of Function

#### (A) **Polynomial function :**

If a function 'f' is called by  $f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \dots + a_{n-1}^{x+a_n}$  where n is a non negative integer and  $a_0, a_1, a_2, \dots, a_n$  are real numbers and  $a_0 \neq 0$ , then f is called a polynomial function of degree n.

#### Note

**(ii)** 

- (I) A polynomial of degree one with no constant term is called an odd linear function. i.e.  $f(x) = ax, \neq 0$ .
  - There are two polynomial functions, satisfying the relation ; f(x), f(1/x). They are :
    - (A)  $f(x) = x^n + 1 \&$
    - (B)  $f(x) = 1 x^n$ , where n is a positive integer.
- (iii) Domain of a polynofunction is R
- (iv) Range of odd degree polynomial is R whereas range of an even degree polynomial is never R.

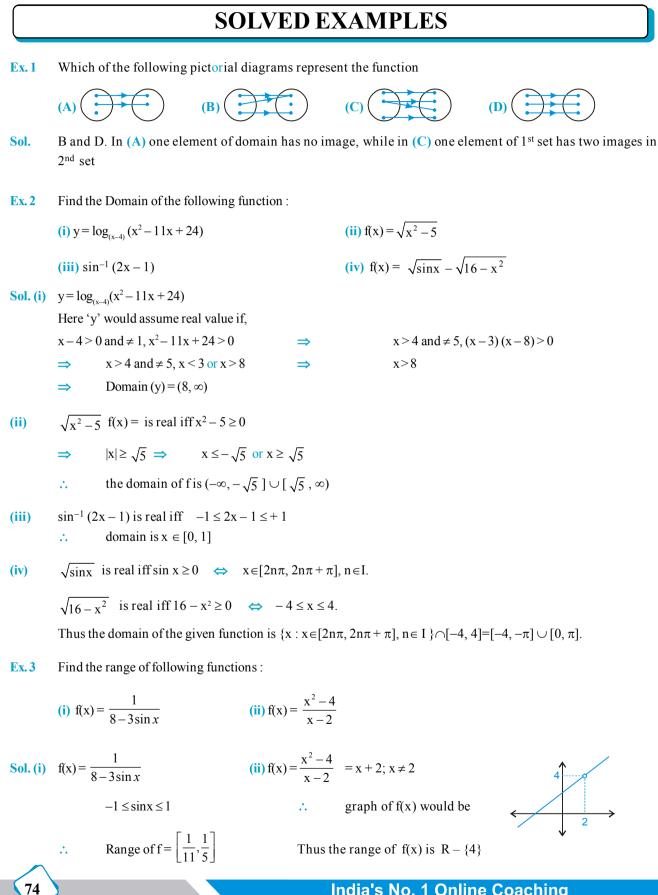
#### (B) Algebric function :

A function 'f' is called an algebric function if it can be constructed using algebric operations (such as addition, subtraction, multiplication, division and taking radicals) straight with polynomials

#### (C) Rational function :

A rational function is a function of the form  $y = f(x) = \frac{g(x)}{h(x)}$ , where g(x) & h(x) are polynomials  $\& h(x) \neq 0$ , **Domain :**  $R - \{x \mid h(x) = 0\}$ 

Any rational function is automatically an algebric function.



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#### MATHS FOR JEE MAIN & ADVANCED

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Single Correct Choice Type Questions

 1. The domain of 
$$f(x) = \sqrt{\frac{1-|x|}{2-|x|}}$$
, is:
 (A)  $(-\infty, \infty) - [-2, 2]$ 
 (B)  $(-\infty, \infty) - [-1, 1]$ 

 (C)  $[-1, 1] \cup (-\infty, -2) \cup (2, \infty)$ 
 (D) none

 2. The domain of the function  $f(x) = \sin^{-1} \left(\frac{1+x^{1}}{2x^{12}}\right) + \sqrt{\sin(\sin x)} + \log_{(3|x|+1)}(x^{2}+1)$ , where {} represents fractional part function, is:
 (A)  $x \in \{1\}$ 
 (B)  $x \in R - \{1, -1\}$ 
 (C)  $x > 3, x \neq 1$ 
 (D) none of these

 3. The domain of the function  $f(x) = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$ , is:
 (A)  $[-2, 0) \cup (0, 1]$ 
 (B)  $(-2, 0) \cup (0, 1]$ 
 (D)  $(-2, 0) \cup [0, 1]$ 

 4. If  $q^{2} - 4 pr = 0, p > 0$ , then the domain of the function  $f(x) = \log(p x^{3} + (p + q) x^{2} + (q + r) x + r)$  is:
 (A)  $R - \left\{-\frac{q}{2p}\right\}$ 
 (B)  $R - \left[(-\infty, -1] \cup \left\{-\frac{q}{2p}\right\}\right]$ 

 (C)  $R - \left[(-\infty, -1) \cap \left\{-\frac{q}{2p}\right\}\right]$ 
 (D) none of these
 (D) none of these

 5. If  $f(x)$  is a polynomial function satisfying the condition  $f(x)$ .  $f(1/x) = f(x) + f(1/x)$  and  $f(2) = 9$  then - (A)  $2f(4) = 3f(6)$ 
 (B)  $14f(1) = f(3)$ 
 (C)  $9f(3) = f(5)$ 
 (D)  $f(10) = f(1)$ 

 6. Domain to function  $\sqrt{\log\{(5x - x^{2})/6\}}$  is .
 (A)  $(2, 3)$ 
 (B)  $[2, 3]$ 
 (C)  $[1, 2]$ 
 (D)  $[1, 3]$ 

 7. Domain and range of  $f(x) = \sqrt{x - 1} + 2\sqrt{3 - x}$  is
 (A)  $(2, 3)$ 
 (B)  $[2, 3]$ 
 (D)  $[1, 5], R: [\sqrt{2}, \sqrt{10}]$ 
 (C)  $[1, -3], R: [\sqrt{2}, \sqrt{10}]$ 
 (D)  $[1, 5], R: [1, \sqrt{3}]$ 
 (D)  $[1, 5], R$ 

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#### MATHS FOR JEE MAIN & ADVANCED

Exercise # 2 [Multiple Correct Choice Type Questions] Part # I 1. Which of the functions defined below are NOT one-one function(s)? (A)  $f(x) = 5(x^2 + 4), (x R)$ **(B)** g(x) = 2x + (1/x)(C)  $h(x) = ln(x^2+x+1), (x R)$ (**D**)  $f(x) = e^{-X}$ Which of the following functions from Z to itself are NOT bijections ? 2. (A)  $f(x) = x^3$ (D)  $f(x) = x^2 + x$ **(B)** f(x) = x + 2(C) f(x) = 2x + 1If  $f(x) = \sin \ell n \left( \frac{\sqrt{4 - x^2}}{1 - x} \right)$ , then 3. (A) domain of f(x) is (-2, 1)**(B)** domain of f(x) is [-1, 1](C) range of f(x) is [-1, 1](**D**) range of f(x) is [-1, 1)The function cot(sinx) -4. (A) is not defined for  $x = (4n+1)\frac{\pi}{2}$ **(B)** is not defined for  $x = n\pi$ (C) lies between -cot1 and cot1 (D) can't lie between -cot1 and cot1 The graph of function f(x) is as shown, adjacently. Then the graph of  $\frac{1}{f(|x|)}$  is -5. y = f(x)b **(A) (B) (C) (D)** b а

## Exercise # 3 Part # I [Matrix Match Type Questions]

Following questions contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with one statement in **Column-II**.

1. Let  $f(x) = \sin^{-1} x$ ,  $g(x) = \cos^{-1} x$  and  $h(x) = \tan^{-1} x$ . For what complete interval of variation of x the following are true. Column – I Column – II

<b>(A)</b>	$f\left(\sqrt{x}\right) + g\left(\sqrt{x}\right) = \pi/2$	<b>(p)</b>	$[0,\infty)$
------------	---	------------	--------------

(B)  $f(x) + g(\sqrt{1-x^2}) = 0$  (q) [0,1]

(C) 
$$g\left(\frac{1-x^2}{1+x^2}\right) = 2h(x)$$
 (r)  $(-\infty, 1)$ 

(D) 
$$h(x)+h(1)=h\left(\frac{1+x}{1-x}\right)$$
 (s) [-1,0]

2.	Column - I		Colum	Column - II	
	<b>(A)</b>	Total number of solution $x^2 - 4 - [x] = 0$ where [] denotes greatest integer function.	<b>(p)</b>	0	
	<b>(B)</b>	Minimum period of $e^{\cos^4\pi x + \cos^2\pi x + x - [x]}$	( <b>q</b> )	1	
	(C)	If A = {(x, y); $y = \frac{1}{x}$ , $x \in R_0$ } and	(r)	2	
	(D)	B = {(x, y) : y = x, x ∈ R} then number of elements in A ∩ B is (are) Number of integers in the domain of $\sqrt{2^x - 3^x} + \log_3 \log_{1/2} x$	(s)	3	

3.	Colur	nn – I	Column – II	
	<b>(A)</b>	The period of the function	<b>(p)</b>	1/2
		$y = \sin (2\pi t + \pi/3) + 2 \sin (3\pi t + \pi/4) + 3 \sin 5\pi t \text{ is}$		
	<b>(B)</b>	$y = {sin (\pi x)}$ is a many one function for $x \in (0, a)$ ,	<b>(q)</b>	8
		where $\{x\}$ denotes fractional part of x, then a may be		
	<b>(C)</b>	The fundamental period of the function		
		$y = \frac{1}{2} \left( \frac{ \sin(\pi/4)x }{\cos(\pi/4)x} + \frac{\sin(\pi/4)x}{ \cos(\pi/4)x } \right) $ is	( <b>r</b> )	2
	<b>(D)</b>	If $f: [0, 2] \rightarrow [0, 2]$ is bijective function defined by $f(x) = ax^2 + bx + c$ , where a, b, c are non-zero real numbers, then $f(2)$ is equal to	<b>(s)</b>	0

## Exercise # 4

[Subjective Type Questions]

1. Find the domain of definitions of the following functions :

(i) 
$$f(x) = \sqrt{3 - 2^x - 2^{1-x}}$$

(ii) 
$$f(x) = (x^2 + x + 1)^{-3/2}$$

(iii) 
$$f(x) = \sqrt{\tan x - \tan^2 x}$$

- (iv)  $f(x) = log_{10}(1 log_{10}(x^2 5x + 16))$
- (v) If  $f(x) = \sqrt{x^2 5x + 4}$  & g(x) = x + 3, then find the domain of  $\frac{f}{g}(x)$

(vi) 
$$f(x) = \frac{1}{[x]} + \log_{1-\{x\}} (x^2 - 3x + 10) + \frac{1}{\sqrt{2 - |x|}} + \frac{1}{\sqrt{\sec(\sin x)}}$$

- 2. Find the range of the following functions :
  - (i) f(x) = 1 |x 2| (ii)  $f(x) = \frac{1}{\sqrt{x 5}}$
  - (iii)  $f(x) = \frac{1}{2 \cos 3x}$ (iv)  $f(x) = \frac{x+2}{x^2 - 8x - 4}$ (v)  $f(x) = \frac{x^2 - 2x + 4}{x^2 + 2x + 4}$ (vi)  $f(x) = 3 \sin \sqrt{\frac{\pi^2}{16} - x^2}$ (vii)  $f(x) = x^4 - 2x^2 + 5$ (viii)  $f(x) = x^3 - 12x$ , where  $x \in [-3, 1]$ (ix)  $f(x) = \sin^2 x + \cos^4 x$
- 3. Let f be a function such that f(3) = 1 and f(3x) = x + f(3x 3) for all x. Then find the value of f(300).

4. Let 
$$f(x) = \frac{9^x}{9^x + 3}$$
 then find the value of the sum  $f\left(\frac{1}{2008}\right) + f\left(\frac{2}{2008}\right) + f\left(\frac{3}{2008}\right) + \dots + f\left(\frac{2007}{2008}\right)$ 

5. Examine whether the following functions are even or odd or neither even nor odd, where [] denotes greatest integer function.

(i) 
$$f(x) = \frac{(1+2^x)^7}{2^x}$$
 (ii)  $f(x) = \frac{\sec x + x^2 - 9}{x \sin x}$ 

(iii) 
$$f(x) = \sqrt{1 + x + x^2} - \sqrt{1 - x + x^2}$$

(iv) 
$$f(x) = \begin{cases} x \mid x \mid, & x \le -1 \\ [1+x] + [1-x], & -1 < x < 1 \\ -x \mid x \mid, & x \ge 1 \end{cases}$$

(v) 
$$f(x) = \frac{2x (\sin x + \tan x)}{2 \left[ \frac{x + 2\pi}{\pi} \right] - 3}$$

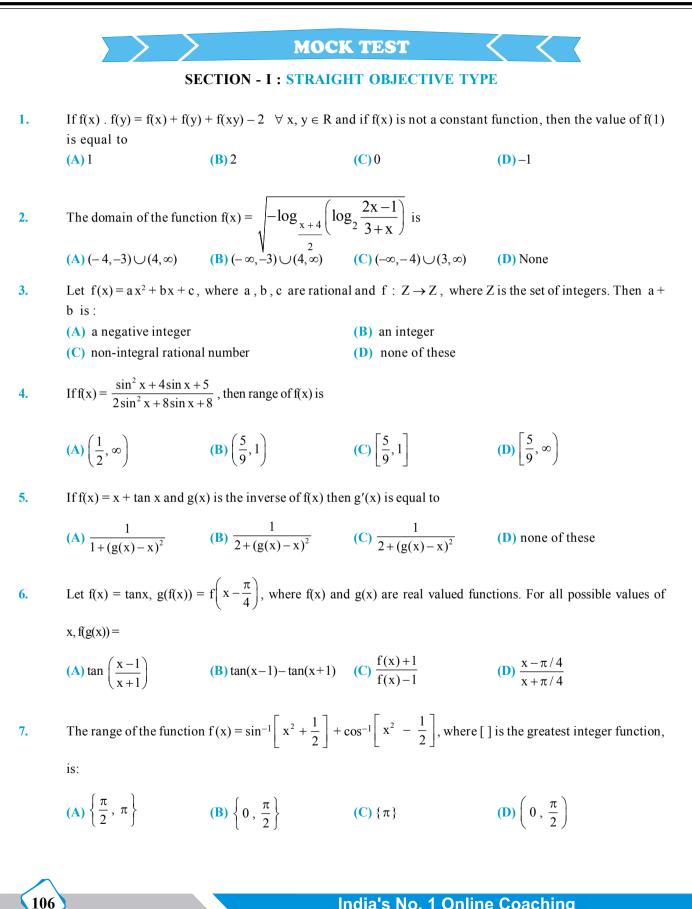
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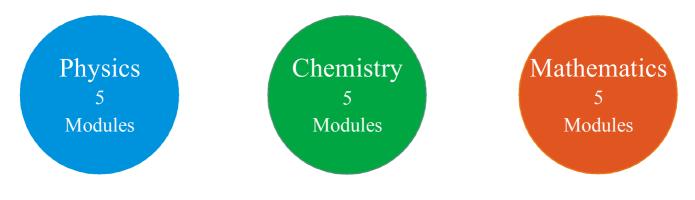
R	Exercise # 5	Part # I > [	Previous Year Questions	s] [AIEEE/JEE-M	IAIN]
1.	Which of the following	ng is not a periodic funct	on-		[AIEEE 2002
	(1) $\sin 2x + \cos x$	(2) $\cos\sqrt{x}$	(3) tan4x	(4) logcos2x	
2.	The period of sin <sup>2</sup> x is	-			[AIEEE 2002
	<b>(1)</b> π/2	<b>(2)</b> π	<b>(3)</b> 3π/2	<b>(4)</b> 2π	
3.	The function $f: R \rightarrow R$	R defined by $f(x) = \sin x$ is	3-		[AIEEE 2002
	<b>(1)</b> into	(2) onto	(3) one-one	(4) many-one	
4.	The range of the func	tion $f(x) = \frac{2+x}{2-x}$ , $x \neq 2$ is			[AIEEE 2002
	(1) R	(2) $R - \{-1\}$	(3) $R - \{1\}$	(4) $R - \{2\}$	
5.	The domain of sin <sup>-1</sup>	$\left[\log_3\left(\frac{x}{3}\right)\right]$			[AIEEE 2002
	(1)[1,9]	(2)[-1,9]	(3)[-9,1]	<b>(4)</b> [-9, -1]	
5.	The function $f(x) = \log \frac{1}{2}$	$g(x + \sqrt{x^2 + 1})$ , is-			[AIEEE 2003
	<ul><li>(1) neither an even no</li><li>(3) an odd function</li></ul>	or an odd function	<ul><li>(2) an even function</li><li>(4) a periodic function</li></ul>		
7.	Domain of definition	of the function $f(x) = \frac{3}{4-}$	$\frac{1}{x^2} + \log_{10} (x^3 - x)$ , is-		[AIEEE 2003
	$(1) (-1, 0) \cup (1, 2) \cup (2, 2$	2,∞)	(2) (1, 2)		
	<b>(3)</b> (−1, 0) ∪ (1, 2)		$(4) (1, 2) \cup (2, \infty)$		
8.	If $f: R \rightarrow R$ satisfies f	(x + y) = f(x) + f(y), for all	$l x, y \in R \text{ and } f(1) = 7$ , then	$\sum_{r=1}^{n} f(r) \text{ is } -$	[AIEEE 2003
	(1) $\frac{7n(n+1)}{2}$	(2) $\frac{7n}{2}$	(3) $\frac{7(n+1)}{2}$	(4) $7n(n+1)$	
).	A function f from the	set of natural numbers to	ntegers defined by $f(n) = \begin{cases} \frac{n}{2} \\ -\frac{n}{2} \end{cases}$	$\frac{-1}{2}$ , when n is odd is $\frac{n}{2}$ , when n is even	- [AIEEE 2003
	(1) neither one-one n		(2) one-one but not		
	(3) onto but not one-	one	(4) one-one and onte	o both	

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## 11<sup>th</sup> Class Modules Chapter Details



#### PHYSICS

#### CHEMISTRY

#### **Module-1**

- 1. Physical World and Units & Dimensions
- 2. Basic Maths & Vector
- **3.** Kinematics

#### Module-2

- 1. Newton's Law of Motion & Friction
- 2. Work, Energy & Power

#### **Module-3**

- 1. Centre of Mass & Collisions
- 2. Rotational Motion
- 3. Gravitation

#### Module-4

- 1. Mechanical Properties of Matter
- 2. Thermal Properties of Matter

#### **Module-5**

- 1. Simple Harmonic Motion
- 2. Wave Motion
- 3. Measurement Error & Experiment

#### Module-1(PC)

- 1. Mole Concept
- 2. Atomic Structure
- 3. Chemical Bonding
- 4. Gaseous State

#### Module-2(PC)

- 1. Thermodynamics
- 2. Thermochemistry
- 3. Chemical Equilibrium
- 4. Ionic Equilibrium

#### Module-3(IC)

- 1. Periodic Table & Its Properties
- 2. Redox Reaction & Equivalent Concepts
- 3. Hydrogen & Its Components
- 4. S-Block

#### Module-4(OC)

- 1. Nomenclature of Organic Compounds
- 2. Isomerism
- 3. General Organic Chemistry

#### Module-5(OC)

- 1. Reaction Mechanism
- 2. Hydrocarbon
- 3. Aromatic Hydrocarbon
- 4. Environmental Chemistry

#### **MATHEMATICS**

#### Module-1

- 1. Basic Maths and Logarithm
- 2. Quadratic Equation
- 3. Sequence and Series

#### Module-2

- 1. Trigonometric Ratio and Identities
- 2. Trigonometric Equation
- **3.** Properties & Solution of Triangle

#### Module-3

- 1. Permutation & Combination
- 2. Binomial Theorum
- 3. Complex Number

#### Module-4

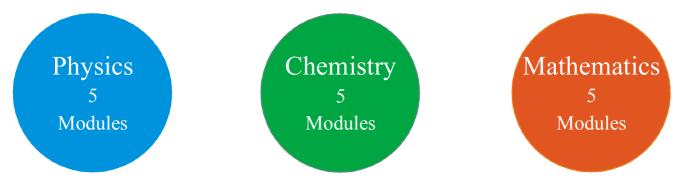
- 1. Straight Line
- 2. Circle
- **3.** Conic Section (Parabola,Ellipse & Hyperbola)

#### Module-5

- 1. Mathematical Induction
- 2. Mathematical Reasoning
- 3. Statistics

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# 12<sup>th</sup> Class Modules Chapter Details



#### PHYSICS

#### **Module-1**

- 1. Electrostatics
- 2. Capacitance

#### Module-2

- 1. Current Electricity
- 2. Magnetic Effect of Current and Magnetism

#### Module-3

- 1. Electromagnetic Induction
- 2. Alternating Current

#### Module-4

- 1. Geometrical Optics
- 2. Wave Optics

#### Module-5

- 1. Modern Physics
- 2. Nuclear Physics
- 3. Solids & Semiconductor Devices
- 4. Electromagnetic Waves
- 5. Principle of Communication

#### CHEMISTRY

#### Module-1(PC)

- 1. Solid State
- 2. Solutions and Colligative Properties
- 3. Electro Chemistry

#### Module-2(PC)

- 1. Chemical Kinetics and Nuclear Chemistry
- 2. Surface Chemistry

#### Module-3(IC)

- 1. Metallurgy
- 2. P-Block
- 3. Transition Elements (d & f block)
- 4. Co-ordination Compound
- **5.** Salt Analysis & Qualitative Analysis

#### Module-4(OC)

- 1. Alkyl Halides & Aryl Halides
- 2. Alcohol, Phenol & Ether
- 3. Carbonyl Compound

#### Module-5(OC)

- 1. Carboxylic Acid & Their Derivatives
- 2. Biomolecules & Polymers
- **3.** Chemistry in Everyday Life

### MATHEMATICS

#### Module-1

- 1. Sets & Relation
- 2. Function
- 3. Inverse Trigonometric Function
- 4. Probability

#### Module-2

- 1. Limit
- 2. Continuity
- 3. Differentiability
- 4. Method of Differentiation

#### Module-3

- 1. Indefinite Integration
- 2. Definite Integration
- 3. Area Under the Curve

#### Module-4

- 1. Application of Derivative
- 2. Matrix
- 3. Determinant

#### Module-5

- 1. Differential Equation
- 2. Vector & 3-Dimensional

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